### STANDARDS FOR EFFICIENT CRYPTOGRAPHY

# SEC 4: Elliptic Curve Qu-Vanstone Implicit Certificate Scheme (ECQV)

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# Contents

1	Introduction			
	1.1	Overview	1	
	1.2	Aim	1	
	1.3	Compliance	1	
	1.4	Document Evolution	1	
	1.5	Intellectual Property	1	
2 Cryp		ptographic Components	3	
	2.1	Security Levels	3	
	2.2	Hash Functions	3	
	2.3	Hashing to Integers Modulo $n$	4	
	2.4	Random Number Generation	4	
	2.5	Elliptic Curve Domain Parameter Generation and Validation	5	
3 ECQV Implicit Certificate Scheme		QV Implicit Certificate Scheme	<b>5</b>	
	3.1	Overview	5	
	3.2	Prerequisites: ECQV Setup	6	
		3.2.1 Certificate Encoding Methods	7	
	3.3	Certificate Request: Cert_Request	8	
	3.4	Certificate Generation Process: Cert_Generate	8	
	3.5	Certificate Public Key Extraction Process: Cert_PK_Extraction $\ldots \ldots \ldots \ldots$	10	
	3.6	Processing the Response to a Cert_Request: Cert_Reception $\ldots \ldots \ldots \ldots \ldots \ldots$	10	
	3.7	$\label{eq:constraint} \mbox{Verifying Explicit ECQV Certificates: $ECQV\_Sig\_Verify$} \ \ldots \ $	11	
	3.8	ECQV Self-Signed Certificate Generation Scheme	12	
	3.9	ECQV Self-Signed Implicit Certificate Public Key Extraction	13	
A	Glossary 1			
	A.1	Terms	15	
	A.2	Acronyms	15	
	A.3	Notation	16	
в	Con	Commentary 17		

$\mathbf{C}$	Representation of ECQV Certificate Structures		
	C.1 Fixed-Length Fields	20	
	C.2 ASN.1 Encodings: Minimal and X.509-Compliant	21	
D	References	29	

# 1 Introduction

### 1.1 Overview

This document specifies the Elliptic Curve Qu-Vanstone implicit certificate scheme (ECQV). The ECQV implicit certificate scheme is intended as a general purpose certificate scheme for applications within computer and communications systems. It is particularly well suited for application environments where resources such as bandwidth, computing power and storage are limited. ECQV provides a more efficient alternative to traditional certificates.

### 1.2 Aim

The aim of this document is to facilitate deployment of the ECQV implicit certificate scheme.

## 1.3 Compliance

Implementations may claim compliance with the cryptographic schemes specified in this document provided the external interface (input and output) to the schemes is equivalent to the interface specified here. Internal computations may be performed as specified here, or may be performed via an equivalent sequence of operations.

Note that this compliance definition implies that conformant implementations must perform all the cryptographic checks included in the scheme specifications in this document. This is important because the checks are essential for security.

## **1.4** Document Evolution

This document will be reviewed at least every five years to ensure it remains up to date with cryptographic advances.

Additional intermittent reviews may also be performed from time-to-time as deemed necessary by the Standards for Efficient Cryptography Group.

External normative standards contain provisions, which, though referenced in this document, constitute provisions of this standard. At the time of publication, the versions indicated were valid. All standards are subject to revision, and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent versions of the standards indicated below.

## 1.5 Intellectual Property

The reader's attention is called to the possibility that compliance with this document may require use of inventions covered by patent rights. By publication of this document, no position is taken with respect to the validity of claims or of any patent rights in connection therewith. The patent holder(s) may have filed with the SECG a statement of willingness to grant a license under these rights on reasonable and nondiscriminatory terms and conditions to applicants desiring to obtain such a license. Additional details may be obtained from the patent holder(s) and from the SECG web site, www.secg.org.

# 2 Cryptographic Components

This section briefly reviews the cryptographic components used in this standard (such as hash functions and random number generators). Much of the content in this section is presented in greater detail in [SEC 1, §3]. Some of the content is also available in ANSI standards, such as [ANSI X9.62].

In this document, the term *Approved* refers to a cryptographic component specified as (or within) a current SECG standard, ANSI X9 standard, or listed in the ANSI X9 registry. The term is used to specify the requirement that primitives such as hash functions and random number generators must conform to existing standards in order to be used with ECQV.

### 2.1 Security Levels

Each of the various cryptographic ingredients to the ECQV scheme has a range of security levels. Security levels are measured in bits. Essentially, each extra bit of security doubles the amount of computation (required by a brute-force attack) to compromise security or, roughly equivalently, halves an adversary's probability of compromising security. At a security level of 128 bits, an adversary would require a computation of roughly  $2^{127}$  or more bit operations to compromise security with probability greater than 50%.

This version of this Standard only recognizes Approved security levels. The currently Approved security levels are the following: 80, 112, 128, 192, and 256 bits.

## 2.2 Hash Functions

Hash functions are used in the ECQV scheme to compute a digest of a certificate that is being generated or verified. Hash functions are also used in verifiably random elliptic curve domain parameter generation and validation, and can also be used in random number generators.

The security level associated with a hash function depends on its application. Where collision resistance is necessary, the security level is at most half the output length (in bits) of the hash function. Where collision resistance is not necessary, the security level is at most the output length (in bits) of the hash function. Collision resistance is generally needed for computing message digests for digital signatures.

The hash functions used by the ECQV scheme shall be Approved hash functions, as Approved in the X9 Registry Item 00003, Secure Hash Standard (SHS). The security level of an Approved hash function is considered to be at most half its output length for the purposes of this Standard. The list of hash functions in [SEC 1, §3.5] are also allowed for use with this standard, excluding SHA-1. The function SHA-1 *must not* be used with this standard, with one exception. Use of SHA-1 is permitted to check verifiably random domain parameters from [SEC 2], when these parameters were generated using SHA-1. See Section 2 of [SEC 2] for more details.

The hash function used for computing the certificate digest shall be a hash function whose security level is at least the security level of the implementation of ECQV. The hash function used for verifiably random elliptic domain parameters shall be a hash function whose security level is the security level of the implementation of the ECQV scheme (with the exception of the verifiably random elliptic curve domain parameters in [SEC 2] which were generated using SHA-1).

It is expected that this standard will be revised to allow the use of SHA-3, once the NIST Cryptographic Hash Algorithm Competition is completed (tentatively by the end of 2012).

### 2.3 Hashing to Integers Modulo n

In many steps of ECQV, an arbitrary length octet string is hashed using a cryptographic hash function H, and then converted to an integer modulo n. This section specifies the resulting function, denoted  $H_n : \{0, 1, \ldots, 255\}^* \rightarrow [0, \ldots, n-1]$ , which maps arbitrary length octet strings to integers modulo n.

#### Input

- 1. An arbitrary length octet string S.
- 2. A hash function H hashing arbitrary length octet strings to bit strings of length hashlen.
- 3. An integer n.

#### Actions

- 1. Compute h = H(S), a bit string of length hashlen bits.
- 2. Derive an integer e from h as follows:
  - 2.1. Set *E* equal to the leftmost  $\lfloor \log_2 n \rfloor$  bits of *h*.
  - 2.2. Convert the bit string E to an octet string E', using the Bit-String-to-Octet-String Conversion specified in [SEC 1].
  - 2.3. Convert the octet string E' to an integer e using the Octet-String-to-Integer Conversion specified in [SEC 1].

**Output** The integer e, in the interval  $[0, \ldots, n-1]$ .

Note that if the domain of H is limited to a maximum size, then the domain of  $H_n$  is constrained in the same way.

### 2.4 Random Number Generation

Within this standard, all random values must be generated using an Approved random number generator. Random number generators must comply with ANSI X9.82 or NIST Special Publication 800-90 [NIST 800-90]. A suitable random number generator is also described in [SEC 1, §3.10].

## 2.5 Elliptic Curve Domain Parameter Generation and Validation

Elliptic curve domain parameter generation and validation shall comply with [SEC 1].

An implementation that uses both ECQV and other ECC algorithms should use a verifiably random base point G, as specified in [SEC 2] (or [ANSI X9.62, A.3.4.1]).

Elliptic curve domain parameters consist of six quantities q, a, b, G, n, and h, which are:

- The field size q.
- The elliptic curve coefficients a and b.
- The base point generator G.
- The order n of the base point generator.
- The cofactor h, which is the number such that hn is the number of points on the elliptic curve.

[SEC 2] specifies elliptic curves which may be used with this standard. Other curves may be used, provided they are generated with the method given in [SEC 1], and the SEED bit string should begin with the ASCII encoding of the string "SEC4:ECQV", and the remainder of the bit string may take an arbitrary value.

# 3 ECQV Implicit Certificate Scheme

This section specifies the Elliptic Curve Qu-Vanstone implicit certificate scheme (ECQV).

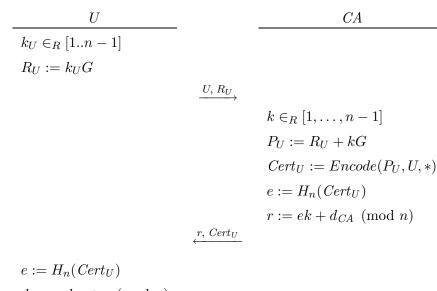
### 3.1 Overview

The implicit certificate scheme is used by three entities – a Certificate Authority CA, a certificate requester U, and a certificate processor V. Requester U will obtain an implicit certificate from CA, certifying U's identity, and allowing V to obtain U's public key.<sup>1</sup>

The implicit certificate scheme consists of six parts. These parts will be described in detail in this section.

- **ECQV\_Setup** In this step CA establishes the elliptic curve domain parameters, a hash function, the certificate encoding format, and all parties have selected a random number generator. CA generates a key pair. All parties must receive authentic copies of CA's public key and domain parameters.
- **Cert\_Request** Requester U must generate a request for a certificate, which is sent to CA. The cryptographic component of the request is a public key, generated using the same procedure as CA uses during ECQV\_Setup.

<sup>&</sup>lt;sup>1</sup>In the language of [RFC 2459], U is the *subject* and V is the *relying party*.



 $e := H_n(Cert_U)$  $d_U := ek_U + r \pmod{n}$  $Q_U := eP_U + Q_{CA}$ 

Figure 1: The ECQV certificate issuance protocol.

- **Cert\_Generate** Upon receiving a certificate request from U, CA confirms U's identity, and creates an implicit certificate. CA sends U the response.
- **Cert\_PK\_Extraction** Given an implicit certificate for user U, the domain parameters, and CA's public key, the public key extraction algorithm computes U's public key.
- **Cert\_Reception** After receiving a response to his certificate request, U ensures the validity of his implicitly certified key pair.
- **ECQV\_Sig\_Verify** Additional information may be provided with an ECQV certificate to form an explicit signature on U's certificate. This step is optional.

Certificate issuance consists of a two step process, where the first step is receiving the certificate request from U, and the second step is to generate a response, containing the certificate. Figure 1 presents a stylized description of the protocol, to which details will be added in subsequent sections.

Some differences between ECQV and traditional certificates with respect to the binding between the user's identity and public key, and the time at which this binding is confirmed by V are discussed in the commentary of Appendix B.

This standard assumes that each entity shall be bound to a unique identifier (e.g., distinguished names). This identity string is denoted U, V and CA for the entities U, V and CA, respectively.

### 3.2 Prerequisites: ECQV Setup

All parties (the certificate authority CA, the certificate owner U, and the certificate processor V) shall establish the following prerequisites in order to use ECQV.

- 1. CA has established a set of EC domain parameters for its use with ECQV consisting of q, a, b, G, (optionally SEED), n and h, along with an indication of the basis used if  $q = 2^m$ . The parameters shall have been generated with an Approved method, such as the method specified in [SEC 1, §3.1], or be Approved parameters, such as those given in [SEC 2]. CA shall have assurance of the validity of the EC domain parameters (especially if CA did not generate them himself, see [SEC 1, §3.1]). The EC domain parameters shall provide at least the desired security level, denoted s.
- 2. CA has selected an Approved hash function with the desired security level s for its use in the ECQV certificate generation process. (See §2.2 for more detailed requirements on the hash function.) Let H denote the hash function chosen, and let *hashlen* denote the length of the output of the hash function. Section 2.3 describes how the output of H is converted to an integer modulo n.
- 3. CA and U have each chosen and initialized an Approved random number generator (as discussed in §2.4) that offers s-bit security. This random number generator shall be used for the generation of private keys created during the certificate request and certificate generation processes.
- 4. CA has obtained an EC key pair  $(d_{CA}, Q_{CA})$  associated with the EC domain parameters established in Item 1 for use during certificate generation. The key pair shall have been generated using the key pair generation primitive in [SEC 1, §3.2.1], using the random number generator established in Item 3. CA shall have assurance of the validity of the key pair and assurance of the possession of the private key (see [SEC 1]).
- 5. The certificate owner U, and the certificate processor V have obtained, in an authentic manner, the elliptic curve domain parameters, the hash function, and  $Q_{CA}$ , CA's public key (established in Item 4). U and V shall have assurance (see [SEC 1]) of:
  - 5.1. the validity of the EC domain parameters,
  - 5.2. the validity of  $Q_{CA}$ , CA's public key, and
  - 5.3. possession of the private key,  $d_{CA}$  by CA.

### 3.2.1 Certificate Encoding Methods

A certificate encoding describes how the information to be included in the certificate should be encoded as an octet string. It also specifies what information must be included, and what information is optional, and any constraints or relationships between portions of the certificate information.

Appendix C describes three certificate encoding methods which may be used with this standard.

**Fixed-length Fields** is a simple, minimalist encoding which puts bandwidth efficiency before all other concerns. The certificate consists of a list of fields, each with a fixed length, and the format is shared amongst all parties. This standard requires only that the public key reconstruction data  $P_U$  be one of the fields, leaving the rest of the format open for implementors.

- Minimal ASN.1 Encoding Scheme is also designed to be bandwidth-efficient. The ASN.1 specification includes basic required fields, and allows extensions. The basic fields suggested in this encoding have fixed length, and therefore may also be used with the simpler, fixed-length format.
- X.509-Compliant ASN.1 Encoding is designed to include sufficient information to allow this ECQV certificate to be encoded as a standard X.509 certificate. Of course, since ECQV does not produce an explicit signature value, the processing logic will differ from an X.509 certificate signed with ECDSA.

# 3.3 Certificate Request: Cert\_Request

The certificate requester U, shall use the process below to generate a certificate request.

### Input

- 1. The elliptic curve domain parameters established by CA as determined in §3.2.
- 2. The hash function H selected by CA as determined in §3.2.
- 3. A string representing U's identity.

### Actions

- 1. Generate an EC key pair  $(k_U, R_U)$  associated with the established elliptic curve domain parameters using the key pair generation primitive specified in [SEC 1, §3.2.1].
- 2. Convert  $R_U$  to the octet string RU using the Elliptic-Curve-Point-to-Octet-String specified in [SEC 1, §2.3].

**Output** The key  $k_U$  and the certificate request (U, RU).

The value RU along with the purported identity U, make up the content of the certificate request. The value  $k_U$  is required in future steps (to compute the private key), and must be kept secret. The certificate request should be sent to CA using a method preserving the data integrity of the message.

# 3.4 Certificate Generation Process: Cert\_Generate

CA shall use the process below to generate a certificate and private key contribution data in response to a Cert\_Request from U. It is assumed that CA has received U, RU in an authenticated manner and has decided to issue a certificate.

#### Input

- 1. The elliptic curve domain parameters established by CA in §3.2.
- 2. The hash function H selected by CA in §3.2.
- 3. CA's private key  $d_{CA}$ .
- 4. A certificate request (U, RU).
- 5. A certificate encoding method with rules for processing as indicated in §3.2.1.
- 6. Additional input fields for the certificate, as described in §3.2.1.

#### Actions

- 1. Convert the octet string RU to an elliptic curve point  $R_U$  using the Octet-String-to-Elliptic-Curve-Point conversion algorithm given in [SEC 1, §2.3.4].
- 2. Validate  $R_U$  using the public key validation technique specified in [SEC 1, §3.2.2]. If the validation primitive outputs 'invalid', output 'invalid' and stop.
- 3. Generate an EC key pair (k, kG) associated with the established elliptic curve domain parameters using the key pair generation primitive specified in [SEC 1, §3.2.1].
- 4. Compute the elliptic curve point  $P_U = R_U + kG$ .
- 5. Convert  $P_U$  to the octet string PU using the Elliptic-Curve-Point-to-Octet-String conversion specified in [SEC 1].
- 6. Call the certificate encoding method with the necessary input fields and the octet string PU as indicated in §3.2.1. If the return value is 'invalid' output 'invalid' and stop, otherwise set the result as the certificate  $Cert_U$ .
- 7. Use the selected hash function to compute  $e = H_n(Cert_U)$ , an integer modulo n.  $H_n$  is defined in §2.3.
- 8. If  $eP_U + Q_{CA} = \mathcal{O}$ , where  $\mathcal{O}$  is the identity element, return to Step 3.
- 9. Compute the integer  $r = ek + d_{CA} \pmod{n}$ .

**Output**  $(r, Cert_U)$ , where r is the private key contribution data, and  $Cert_U$  is the certificate.

The response from CA may be made public. Additionally, it may be communicated over an insecure channel, as U may verify that the received information is valid, using the Cert\_Reception procedure of Section 3.6.

### 3.5 Certificate Public Key Extraction Process: Cert\_PK\_Extraction

The public key bound to the certificate is derived from the certificate using CA's public key, and is recovered using the following process. This step does not require any secret information, and may be performed by any user who knows  $Cert_U$ , and the public parameters output by ECQV\_Setup.

### Input

- 1. The elliptic curve domain parameters established by CA in §3.2.
- 2. The hash function H selected by CA in §3.2.
- 3. CA's public key  $Q_{CA}$  as determined in §3.2.
- 4. The certificate  $Cert_U$ .

#### Actions

- 1. Decode the certificate  $Cert_U$  according to the certificate decoding methods and rules. If the return value is 'invalid' then output 'invalid' and stop, otherwise an octet string PU will be returned.
- 2. Convert PU to a point  $P_U$  using the Octet-String-to-Elliptic-Curve-Point conversion specified in [SEC 1, §2.3].
- 3. Validate  $P_U$  using the public key validation technique specified in [SEC 1, §3.2.2]. If the validation primitive outputs 'invalid', output 'invalid' and stop.
- 4. Use the selected hash function to compute  $e = H_n(Cert_U)$ , an integer modulo n.  $H_n$  is defined in §2.3.
- 5. Compute the point  $Q_U = eP_U + Q_{CA}$ .

**Output** Either 'invalid' or a public key  $Q_U$ .

Note that if Cert\_PK\_Extraction outputs  $Q_U$ , then  $Q_U$  is valid in the sense of [SEC 1]; it is a point of order n on the input EC domain parameters.

## 3.6 Processing the Response to a Cert\_Request: Cert\_Reception

This routine validates the contents of an implicit certificate and the private key contribution data issued by CA. The output of Cert\_PK\_Extraction, and the private key computed here form a key pair. Recall that Cert\_PK\_Extraction performs the public key validity check from [SEC 1], so it is not repeated here. The certificate requester U shall use the process below upon receipt of their certificate request response to compute the private key for the public key output by Cert\_PK\_Extraction, and validate the key pair.

#### Input

- 1. The elliptic curve domain parameters established by CA in §3.2.
- 2. The hash function H selected by CA in §3.2.
- 3. The private value  $k_U$  generated by U in §3.3.
- 4. The output of Cert\_Generate: the certificate  $Cert_U$  and the private key contribution data, an integer r.
- 5. The public key  $Q_U$ , as output by Cert\_PK\_Extraction.

#### Actions

- 1. Use the selected hash function to compute  $e = H_n(Cert_U)$ , an integer modulo n.
- 2. Compute the private key  $d_U = r + ek_U \pmod{n}$ .
- 3. Compute  $Q'_U = d_U G$ .

**Output** 'valid' and  $d_U$  if  $Q_U$  is equal to  $Q'_U$ , and 'invalid' otherwise.

## 3.7 Verifying Explicit ECQV Certificates: ECQV\_Sig\_Verify

This routine is used in applications where V would like to verify that  $Cert_U$  is a valid certificate issued by CA before performing subsequent cryptographic operations using U's public key, extracted from  $Cert_U$ . This prevents V from using an invalid certificate (possibly created by someone other than CA). This feature allows V to explicitly verify the binding between U's identity and the public key extracted from  $Cert_U$ . This routine is not applicable to self-signed certificates. The inputs to ECQV\_Sig\_Verify will typically be sent from U to V.

#### Input

- 1. The elliptic curve domain parameters established by CA in §3.2.
- 2. The hash function H selected by CA in §3.2.
- 3. CA's public key,  $Q_{CA}$ .
- 4. The certificate request octet string RU.
- 5. The output of Cert\_Generate: the certificate  $Cert_U$  and the private key contribution data, an integer r.

### Actions

- 1. Convert RU to a point  $R_U$  using the Octet-String-to-Elliptic-Curve-Point conversion specified in [SEC 1, §2.3].
- 2. Decode the certificate  $Cert_U$  according to the certificate decoding methods and rules. If the return value is 'invalid' then output 'invalid' and stop, otherwise an octet string PU will be returned.
- 3. Convert PU to a point  $P_U$  using the Octet-String-to-Elliptic-Curve-Point conversion specified in [SEC 1, §2.3].
- 4. Validate  $P_U$  and  $R_U$  using the public key validation technique specified in [SEC 1, §3.2.2]. If the validation primitive outputs 'invalid' for either point, output 'invalid' and stop.
- 5. Use the selected hash function to compute  $e = H_n(Cert_U)$ , an integer modulo n.
- 6. Compute and compare the points rG and  $Q_{CA} + e(P_U R_U)$ .

**Output** 'valid' if the points compared in Step 6 are equal, and 'invalid' otherwise.

# 3.8 ECQV Self-Signed Certificate Generation Scheme

This section specifies the scheme for generating self-signed implicit certificates. In the self-signed certificate generation scheme, the user U generates the certificate request and performs the actions of CA as well, but sets CA's key pair to  $(0, \mathcal{O})$  (the private key is zero and the public key is the identity element of the elliptic curve group). The certificate must indicate that it is a self-signed certificate to allow the public key to be extracted correctly.

 ${\cal U}$  shall execute the following steps to create a self-signed implicit certificate.

### Input

- 1. The elliptic curve domain parameters, as determined in §3.2.
- 2. The hash function H, as determined in §3.2.
- 3. A certificate encoding method with rules for processing as indicated in  $\S3.2.1$ .
- 4. Additional input fields for the certificate, as described in  $\S3.2.1$ .

### Actions

1. Use the key pair generation primitive specified in [SEC 1, §3.2.1] to generate a key pair  $(k_U, P_U)$  associated with the established domain parameters.

- 2. Convert the elliptic curve point  $P_U$  to the octet string PU using the Elliptic-Curve-Point-to-Octet-String conversion algorithm given in [SEC 1, §2.3].
- 3. Call the certificate encoding method with the necessary input fields and the octet string PU as indicated in §3.2.1. If the return value is 'invalid' output 'invalid' and stop, otherwise set the result as the certificate  $Cert_U$ .  $Cert_U$  must indicate that it is self-signed.
- 4. Use the selected hash function to compute  $e = H_n(Cert_U)$ , an integer modulo n.
- 5. Compute the private key  $d_U = ek_U \pmod{n}$ .

**Output** If any of the above verifications has failed, then output 'invalid' and stop; otherwise, output 'valid', and  $Cert_U$  as U's self-signed implicit certificate, and  $d_U$  as the corresponding private key.

### 3.9 ECQV Self-Signed Implicit Certificate Public Key Extraction

This section specifies the scheme for extracting the public key from a self-signed implicit certificate. As with the process for generating a self-signed certificate, extracting the public key follows the extraction process but with the CA key pair set to  $(0, \mathcal{O})$ .

#### Input

- 1. The elliptic curve domain parameters, as determined in §3.2.
- 2. The hash function H, as determined in §3.2.
- 3. A certificate encoding method with rules for processing as indicated in §3.2.1.
- 4. The self-signed certificate  $Cert_U$  output by the process in §3.8.

#### Actions

- 1. Decode the certificate  $Cert_U$  according to the certificate decoding methods and rules. If the return value is 'invalid' then output 'invalid' and stop, otherwise an octet string PU will be returned. Ensure that  $Cert_U$  is self-signed, by checking the appropriate field. If  $Cert_U$  is not self-signed, output 'invalid' and stop.
- 2. Convert PU to a point  $P_U$  using the Octet-String-to-Elliptic-Curve-Point conversion specified in [SEC 1, §2.3].
- 3. Validate  $P_U$  using the public key validation technique specified in [SEC 1, §3.2.2]. If the validation primitive outputs 'invalid', output 'invalid' and stop.
- 4. Use the selected hash function to compute  $e = H_n(Cert_U)$ , an integer modulo n.
- 5. Compute the point  $Q_U = eP_U$ .

**Output:** If none of the above steps has output 'invalid' output the point  $Q_U$  as the public key corresponding to  $Cert_U$ . Otherwise output 'invalid'.

# A Glossary

This section provides a glossary to the terms, acronyms, and notation used in this document.

Please refer to the glossary of SEC1 [SEC 1] for any term, acronym, or notation not specified in this section.

# A.1 Terms

Terms used in this document include:

(traditional) certificate	Information including the public key and identity of an en- tity, cryptographically signed by a Certificate Authority. See "certificate" in Appendix A.1 of SEC1 [SEC 1].
implicit certificate	Information including public-key reconstruction data and the identity of an entity that constitute the certificate of that entity.
public-key reconstruction data	An elliptic curve point contained in the implicit certificate, from which any party with access to $CA$ 's public key can reconstruct the public key associated with the certificate.
private-key reconstruc- tion data	Value computed by a $CA$ during the creation of an implicit certificate. This may be a public value that allows the cer- tificate requester to compute his private key. Also called the CA contribution to the private key.
to-be-signed-certificate data	Data to be included in a certificate or implicit certificate. This data includes the identity of the certified entity, but may also include other data, such as the intended use of the public key, the serial number of the certificate, and the validity period of the certificate. The exact form of this data depends on the certificate encoding being used.

# A.2 Acronyms

The acronyms used in this document denote:

ECQV	Short for the Elliptic Curve Qu-Vanstone implicit certificate scheme described in Section 3.
CA	The certificate authority.

# A.3 Notation

The notation adopted in this document is:

$Cert_U$	Implicit certificate for user $U$ .
$P_U$	Public reconstruction data for user $U$ , an elliptic curve point.
PU	Public reconstruction data for user $U$ , encoded as an octet string.
$I_U$	To-be-signed-certificate data, non-cryptographic information about user $U$ , such as their identity and the validity of the cer- tificate
$R_U$	Certificate request value created by $U$ , an elliptic curve point.
RU	Certificate request value created by $U$ , encoded as an octet string.
$Q_U$	User $U$ 's public key.
$egin{array}{c} Q_U \ Q_{CA} \end{array}$	CA's public key.

# **B** Commentary

The aim of this section is to supply implementers with relevant guidance. However, this section does not attempt to provide exhaustive information but rather focuses on giving basic information and including pointers to references which contain additional material. Furthermore, this section concentrates on supplying information specific to implicit certificates. Extended commentary on ECC in general – addressing issues like parameter selection and implementation of elliptic curve arithmetic – can be found in [SEC 1, Appendix B]. This section provides a commentary on the ECQV implicit certificate scheme. It discusses properties specific to this scheme, and some security considerations specific to ECQV certificates.

**Binding of Identity and Key Pair** With all digital certificates that bind an identity to a public key, there are two aspects of certification to consider. The first is the binding between the user identity and his public key. The second is assurance that the user has knowledge of his private key.

The implicit certificate generation algorithm yields a static public key purportedly bound to U (i.e., purportedly issued by CA). When explicit ECQV verification is not used, confirmation that this public key is genuinely bound to U is only obtained after use of the corresponding key pair (e.g., via execution of an authenticated key agreement scheme involving this key pair). Thus, with implicit certificates, the binding of an entity and its public key and knowledge of the private key are verified in unison, during key usage.

This situation differs from ordinary certificates (e.g., X.509 certificates), where the binding between U and his public key is confirmed by verifying CA's signature in the certificate. Proof that U knows the corresponding private key is only obtained during cryptographic usage of the key pair. Some certificate issuance protocols require U to prove knowledge of his private key to CA. In this case, another user V has indirect assurance that U knows his private key if the certificate is valid (since V trusts CA to perform this check when the certificate was issued).

Using the explicit signature feature of the ECQV scheme allows V to confirm the binding between U and his public key prior to usage. As such, an implicit certificate augmented with some additional information form an explicit signature that may be used in contexts where the assurances of ordinary certificate processing are desirable.

**Key Pair Generation** With traditional certificates, key pair generation and certificate issuance are two independent processes. A user can present an arbitrary public key to a CA for certification. In ECQV the situation is somewhat different. When a user requests an implicit certificate for a public key from a CA, this public key (and the private key) is a randomized result of the joint computation by the user and the CA.

This has the direct consequence that once an ECQV implicit certificate is issued, one cannot get another ECQV implicit certificate for the same public key from a different CA. It is possible however, to obtain a traditional certificate on an ECQV public key.

Another difference is that the private key is not generated by one party, but computed jointly by the CA and the requester. Because of this, with low probability the private key may be zero, and the CA must check for this in Step 8 of Cert\_Generate. It might also happen that the key pair jointly generated for the requester equals the key pair of a previously issued certificate. It is recommended that the CA check to ensure that the public key associated to an implicit certificate has not previously been issued.

**Domain Parameters Associated to a Key Pair** Recall that U's public key associated with an ECQV implicit certificate  $Cert_U$  is computed as  $Q_U = H_n(Cert_U) \cdot P_U + Q_{CA}$ . Since  $Q_{CA}$  is defined over a certain elliptic curve, this computation must be performed over the same curve. This means that the constructed user's public key is defined over the same elliptic curve as CA's public key. Hence, the key pair certified by an ECQV implicit certificate has to be defined over the same elliptic curve parameters as used by CA. In particular the security level of the certified key pair is the same as CA's key pair.

**Efficiency** An advantage of implicit certificates is that since they contain only the public reconstruction data instead of the subject's public key and the CA's signature, they may be smaller than traditional certificates. We specify three certificate formats in Section C.

With respect to computational efficiency, we note that the public key extraction step may be combined with other operations. For instance, if a protocol requires computation of  $zQ_U$  where  $Q_U$  is a public key implicitly certified by  $Cert_U$  then instead of computing  $Q_U = H(Cert_U)P_U + Q_{CA}$ followed by  $zQ_U$  (two scalar multiplications) it may be faster to compute  $zH(Cert_U)P_U + zQ_{CA}$ (using fast scalar multiplication/exponentiation techniques).

**Demonstrating Knowledge of the Secret Key During Certificate Issuance** When an entity U requests a traditional certificate for a public key, U should prove to the CA it knows the corresponding private key. This is to prevent U from choosing an arbitrary public key, that may already belong to another user, and have it certified. This situation is clearly undesirable (and may even lead to security problems).

With implicit certificates this proof is unnecessary, as there is no public key before the certificate is issued. Further, U has no control over the final value of his public key, due to the CA's contribution, making it impossible for U to cause the confusion described above.

**Unforgeability** Unlike traditional certificates, an implicit certificate does not contain a digital signature. In fact, one could simply choose an arbitrary identity I and a random value to form a certificate. Together with the public key of a CA, this generates a public key for the entity identified by I. However, if one constructs an implicit certificate in such a way, i.e., without interacting with the CA, it is infeasible to compute the private key that corresponds to the public key generated by the certificate. See [BGV01] for a security analysis of ECQV.

Another difference between traditional certificates and implicit certificates is that when presented with a valid traditional certificate, one knows that the certificate belongs to *someone*. A valid certificate containing the certificate data string  $I_U$  is a proof that the CA signed this certificate for U, and also that U knows the private key corresponding to the public key included in the certificate. One does not have this guarantee with implicit certificates. It is trivially possible to construct an implicit certificate  $Cert_U$  such that the private key corresponding to the public key computed as  $Q_U = H_n(Cert_U) \cdot P_U + Q_{CA}$  is unknown.

This fact suggests a denial-of-service type attack, where a party V is flooded with protocol requests using "fake" implicit certificates. The fact that the private key of the fake certificate is unknown is only revealed after V has performed most of the protocol. Of course, a similar attack can be launched in a system using traditional certificates. In this case, the attacker would flood a party with various certificates belonging to other entities. The certificates are valid, but the attacker does not know the private key of the corresponding public key. Nevertheless, if this type of attack on ECQV is a concern, the explicit verification feature of ECQV may be used (see Section 3.7).

The values required for explicit verification are sometimes called a *signature*, since CA's response  $(r, Cert_U)$  is similar to a Schnorr signature created by CA [Sch91], but with a modified signing equation. This variant of Schnorr's signature scheme may be proven secure using a theorem of Abdalla et al. [AABN02], which generalizes the proof techniques of Pointcheval and Stern [PS96].

**Composability** Composition of ECQV with other primitives requires care. By *composed*, we mean using the implicit public key computed with Cert\_PK\_Extraction with another cryptographic primitive. For instance, using a validated ECQV public key as an ECDSA verification key is known to be secure, provided the signed message is not the same as the certificate, i.e., verifiers should not accept a signature from U on the message  $Cert_U$ . See the analysis of ECQV-certified ECDSA by Brown et al. [BCV09]. Composition of ECQV with other primitives from SEC1 such as key-agreement (ECMQV) and encryption (ECIES) have not been formally analyzed in the literature, however no attacks against these compositions are known either.

# C Representation of ECQV Certificate Structures

Here we specify the encoding and decoding operation primitives that may be used to generate ECQV certificates and process ECQV certificates. We provide three options, a simple fixed-length encoding, a minimal ASN.1 encoding, and an X.509-like ASN.1 encoding. Implicit certificates are specifically defined to reduce bandwidth requirements therefore creating highly extensible and verbose encoding methods are counterproductive to the goals of the scheme. The schemes defined indicate rules associated with elements of the certificate. These rules typically enforce public key infrastructure policies, such as key-usage, validity periods, issuer identifier, subject identifier, etc.

Optionally, applications may re-encode certificates that have been issued in one encoding to another encoding, provided the encoding used by the CA is designated as the canonical encoding. Verifiers must reverse the re-encoding of the received certificate to the canonical encoding before hashing the certificate in Cert\_PK\_Extraction (§3.5), and in ECQV\_Sig\_Verify (§3.7). The final version of this standard will specify how implementations should indicate the canonical encoding.

# C.1 Fixed-Length Fields

The fixed-length fixed-fields encoding and decoding methods uses concatenation of a fixed and agreed number of octet string fields of fixed length. We do not specify fields to be included. This is left to individual implementations. The fixed-length fixed-fields encoding may easily be customized to a particular application to include only those fields which are necessary for the application, leading to a compact certificate. The fields given in the minimal encoding scheme of  $\S C.2$  may also be used with this encoding, as they have fixed length.

### Prerequisites

The entities using this method shall agree on the following choices for the method:

- A number of fields f.
- The octet length of each field, denoted  $len_i$  for i = 1 to f.
- The index i, of the field that will contain the encoded public key reconstruction data (an octet encoded elliptic curve point).
- Validity rules for each of the field elements.

## Fixed-Length Fixed-Fields Encoding Operation

**Input** The input to the encoding operation is:

- Octet strings  $F_1, F_2, \ldots, F_f$ ,
- $F_i = PU$  an octet encoded elliptic curve point for some *i* in  $[1, \ldots, f]$ .

Actions Compute the ECQV certificate as follows:

- Verify each field  $F_i$  is of length  $len_i$  and if not return 'invalid' and stop.
- Validate that each field satisfies the rules specified in the prerequisites, and if any fail return 'invalid' and stop.
- Create the certificate  $Cert = F_1||F_2||\dots||F_f$ .

**Output** The octet string *Cert* or 'invalid.'

### Fixed-Length Fixed-Fields Decoding Operation

**Input** The input to the encoding operation is:

• The certificate *Cert*.

Actions Decode the certificate as follows:

- Verify that the length of *Cert* is equal to  $len_1 + len_2 + \ldots + len_f$ , if not return 'invalid' and stop.
- Parse the certificate into pre-defined length segments  $Cert = F_1||F_2||...||F_f$ .
- Validate that each field satisfies the rules specified in the prerequisites, and if any fail return 'invalid' and stop.

**Output** The field values  $F_1, \ldots, F_f$ , where one of  $F_i = PU$ , an octet-encoded public key reconstruction value.

# C.2 ASN.1 Encodings: Minimal and X.509-Compliant

We provide two ASN.1 encodings. The first is called the minimal encoding scheme (MES). The MES is designed to be as compact as possible, and is a list of basic certificate fields which are common to many applications. The fields in the MES are fixed length, which means the fields in this format may be encoded using the simple encoding of §C.1, however, for greater portability it may also be encoded with the ASN.1 syntax provided in this section.

The second format we describe is an X.509 compliant format. The X.509 certificate format is given in [RFC 2459]. This format is larger, but allows ECQV certificates to be encoded and parsed as X.509 certificates. We also give a mapping between the two formats which allows a MES encoded ECQV certificate to be re-encoded as an X.509 encoded ECQV certificate. Re-encoding in the other direction is possible, however, since the X.509 format allows more information to be stored in the certificate, some information may be lost when re-encoding. Put another way, the mapping of ECQV certificates from MES to X.509 encodings is one to many, because of the additional fields in the X.509 encoding. The ASN.1 module specifying the encodings of ECQV certificates is given in below.

In both ASN.1 encoded formats, to indicate that a certificate is self-signed, the issuerID (or issuer) field should be set to zero.

-- --------------- ECQV certificate format: minimal encoding scheme (MES) -- ASN.1 is used to describe the format, but these fields -- could also be used with the fixed-length field encoding. -- The times are represented using UNIX time, i.e., # of seconds -- since the unix epoch: http://en.wikipedia.org/wiki/Unix\_time -- The validFrom field uses 40-bits to avoid problems in -- 2038 (when 32-bits won't be enough). -- -----ANSI-X9-YY{iso(1) member-body(2) us(840) 10045 module(0) 2} DEFINITIONS EXPLICIT TAGS ::= BEGIN ansi-X9-YY OBJECT-IDENTIFIER ::= {iso(1) member-body(2) us(840) 10045} ECQVCertificate ::= SEQUENCE { type MESType DEFAULT t1, -- one byte, see below serialNumber OCTET STRING (SIZE (8)), curve Curve, -- named curve, see below hash Hash, issuerID OCTET STRING (SIZE (8)), validFrom OCTET STRING (SIZE (5)), -- 40-bit Unix time validDuration OCTET STRING (SIZE (4)),-- 32-bit # of seconds subjectID OCTET STRING (SIZE (8)), usage KeyUsage, -- one byte, described below pubKey OCTET STRING, . . . , -- Extensions: explicitSig[1] ECQV-sig-value OPTIONAL, algorithm[2] AlgorithmIdentifier OPTIONAL, email[3] IA5String (SIZE (0..128)) OPTIONAL } -- Notes: -- \* The 32-bit # of seconds allows certs to be valid for up to 136 years. If validDuration =  $2^{32} - 1$  (the maximal unsigned 32-bit integer) then the certificate is valid forever, i.e., it has no expiry -- \* The subjectID could be a MAC address (they are 48-bits, and the OUI is least significant three octets) ---- \* If the explicitSig extention is present type MUST be v3 or v4. If

```
extensions other than explicitSig are used, type MUST be v2, v4 or v6, and
__
    if type is v2, v4 or v6, both extensions (algorithm and email) MUST be
___
    present.
__
-- * The size of pubKey varies depending on the curve chosen. Note this is only
    the public key reconstruction data.
___
-- * The algorithm extension is used to specify what the user's public key will
    be used for (e.g., ecdsa-with-sha256)
___
-- * The email extension can be null terminated instead of fixed length, and be
-- a maximum of 128 bytes.
-- * The total size is: 37 bytes + |pubKey| + |extensions|
MESType ::= INTEGER {
   t1(0), -- type 1: implicit with no extension(s),
   t2(1), -- type 2: implicit with extension(s)
   t3(2), -- type 3: explicit with no extension(s)
   t4(3), -- type 4: explicit with extension(s)
}
-- Curves for use with ECQV. Includes all curves listed in SEC2.
Curve ::= INTEGER { secp192k1(0), secp192r1(1), secp224k1(2),
secp224r1(3), secp256k1(4), secp256r1(5), secp384r1(6), secp512r1(7),
sect163k1(8), sect163r1(9), sect233k1(10), sect233r1(11), sect239k1(12),
sect283k1(13), sect283r1(14), sect409k1(15), sect409r1(16), sect571k1(17),
sect571r1(18) }
Hash ::= INTEGER { id-sha224(0), id-sha256(1), id-sha384(2),
id-sha512(3) }
KeyUsage ::= BIT STRING { -- see rfc 3280 for more details
                       (0),
digitalSignature
       nonRepudiation
                               (1),
       keyEncipherment
                               (2),
       dataEncipherment
                               (3),
       keyAgreement
                               (4),
       keyCertSign
                               (5),
       cRLSign
                               (6),
       encipherOnly
                               (7),
       decipherOnly
                               (8)
}
-- -----
-- X.509 ECQV certificate structures
__ ____
-- This certificate format is compliant with x.509, and corresponds to a MES
-- encoded ECQV certificate The descripition of X.509 is here:
-- http://www.ietf.org/rfc/rfc3280.txt
-- -----
```

```
-- OIDs required: ecqv-with-sha224, ecqv-with-sha256, ecqv-with-sha384,
-- ecqv-with-sha512, ecqv-with-aes-mmo
-- signatureAlgorithm is the CA's signing algorithm (always ECQV-something).
-- The explicitSig value is only for explicit ECQV certificates.
ECQV-X509-Certificate ::= SEQUENCE {
   tbsCertificate
                      TBSCertificate,
    signatureAlgorithm AlgorithmIdentifier,
    explicitSig ECQV-sig-value OPTIONAL
}
-- This is the certificate data to be signed. Notes:
-- * versions have the same interpretation as in rfc3280
-- * signature denotes the CA's signing algorithm. MUST be the same as
__
    signatureAlgorithm above. The valid algorithm identifiers are the
    OIDs listed above.
__
-- * subjectPublicKeyInfo contains the public key reconstruction data
-- * extensions can be any valid X.509 extension. If present, version
   MUST be v3
___
TBSCertificate ::= SEQUENCE {
   version
                   Version DEFAULT v1,
   serialNumber CertificateSerialNumber,
   signature
                   AlgorithmIdentifier,
                   Name, -- corresponds to issuerID in minimal encoding
   issuer
                    Validity,-- as in rfc3280 (X.509)
   validity
   subject
                   Name, -- corresponds to subjectID in minimal encoding
    subjectPublicKeyInfo SubjectPublicKeyInfo,
    extensions[3] Extensions OPTIONAL
}
Version ::= INTEGER { v1(0), v2(1), v3(2) }
CertificateSerialNumber ::= INTEGER
Validity ::= SEQUENCE {
    notBefore
                   Time,
    notAfter
                   Time
}
-- This is the way time is done in PKIX (from rfc 3280). Some details of the
-- time conversion are given in the MES->X.509 mapping below.
Time ::= CHOICE {
    utcTime
                   UTCTime,
    generalTime GeneralizedTime
}
```

```
UniqueIdentifier ::= BIT STRING
-- The AlgorithmIdentifier specifies the algorithm the user will use their
-- keypair for. Note the parameters MUST be empty, since the user's key
-- inherits the paramters from the CA (No equivalent in minimal encoding.)
-- Note that subjectPublicKey is the public key reconstruction data P_U,
-- NOT the public key.
SubjectPublicKeyInfo ::= SEQUENCE {
    algorithm
                         AlgorithmIdentifier,
    subjectPublicKey
                         ECPoint
}
Extensions ::= SEQUENCE SIZE (1..MAX) OF Extension
Extension ::= SEQUENCE {
    extnID OBJECT IDENTIFIER,
    critical BOOLEAN DEFAULT FALSE,
    extnValue OCTET STRING
}
-- This is similar to the x.509 ASN.1 encoding of the algorithm field,
-- but drops the parameters field, since the subjects' public key
-- can only be used with the same parameters as the CA.
AlgorithmIdentifier ::= SEQUENCE {
    algorithm
                       OBJECT IDENTIFIER,
}
-- The ECQV-sig-value is only required when the certificate contains the
-- explicit ECQV cert. The signature value is r, the CA response to a request.
-- The point value is R_U, the certificate request point (ECPoint is defined in
-- SEC1).
ECQV-sig-value ::= SEQUENCE {
version [0] Version DEFAULT v2,
signature[1] OCTET STRING OPTIONAL,
point[2] ECPoint OPTIONAL
}
-- The following elliptic curve structures are included for completeness.
-- Named Elliptic Curves in ANSI X9.62.
ansi-X9-62 OBJECT IDENTIFIER ::= { iso(1) member-body(2) us(840) 10045 }
ellipticCurve OBJECT IDENTIFIER ::= { ansi-X9-62 curves(3) }
-- Elliptic Curve parameters may be specified explicitly, specified implicitly
-- through a "named curve", or inherited from the CA. Note that explicitly
-- specified parameters are not supported by the MES therefore, X.509 ECQV
```

```
-- certificates with explicitly specified parameters cannot be encoded using the
-- MES format.
EcpkParameters ::= CHOICE {
   ecParameters ECParameters,
   namedCurve OBJECT IDENTIFIER,
   implicitlyCA NULL
}
-- Elliptic curve parameters
ECParameters ::= SEQUENCE {
   version ECPVer,
   fieldID FieldID,
   curve Curve,
   base
           ECPoint, -- Base point G
   order INTEGER,
                                   -- Order n of the base point
     cofactor INTEGER OPTIONAL -- The integer h = \#E(Fq)/n
  }
FieldID ::= SEQUENCE {
   fieldType OBJECT IDENTIFIER,
   parameters ANY DEFINED BY fieldType
}
ECPVer ::= INTEGER {ecpVer1(1)}
ECPoint ::= OCTET STRING
_____
-- X.501 Name type
_____
Name ::= CHOICE {
   rdnSeq RDNSequence
}
RDNSequence ::= SEQUENCE OF RelativeDistinguishedName
RelativeDistinguishedName ::=
   SET OF AttributeTypeAndValue
AttributeTypeAndValue ::= SEQUENCE {
            OBJECT IDENTIFIER,
   type
   value
            [0] EXPLICIT ANY DEFINED BY type
}
DirectoryString ::= CHOICE {
   teletexString TeletexString (SIZE (1..MAX)),
   printableString PrintableString (SIZE (1..MAX)),
```

```
universalString UniversalString (SIZE (1..MAX)),
utf8String UTF8String (SIZE (1..MAX)),
bmpString BMPString (SIZE (1..MAX))
}
-- _____
-- MES and X.509 field equivalence
-- Here we describe how to encode a MES encoded certificate as an X.509
-- certificate (and vice-versa, the mapping given here is 1:1. Of course, the
-- X.509 may contain additional fields.) Field by field correspondence is given.
-- Note there is no loss of time precision since X.509 times MUST include
-- seconds and MUST NOT include fractional seconds.
-- _____
-- MES field X.509 field
-- -----
                   _____
-- type tbsCertificate.version = v3
-- Note: version must always be v3, since the KeyUsage field is a basic field
-- in MES, but extension in X.509
-- serialNumber tbsCertificate.serialNumber
         tbsCertificate.signature.ecParams.curve
-- curve
-- hash
                given by the name of the AlgorithmIdentifier
                tbsCertificate.signature
-- issuerID tbsCertificate.issuerID (note it uses the Name type)
-- validFrom tbsCertificate.Validity
-- validDuration
-- Note: Validity.notBefore = validFrom,
-- Validity.notAfter = notBefore + validDuration.
-- Also note: MES encodes time as unix time, and X.509 uses the Time type.
-- If validDuration = 2<sup>32</sup> -1 (the maximal unsigned 32-bit integer) then the
-- certificate is valid forever, i.e., it has no expiry.
-- subjectID tbsCertificate.subject (note it uses the Name type)
-- usage
                tbsCertificate.usage (Note: this is an X.509 extension)
-- pubKey
                tbsCertificate.subjectPublicKeyInfo.subjectPublicKey
-- ***
                signatureAlgorithm = ecqv-with-hash
-- Note: both signatureAlgorithm and tbsCertificate.signature should be the same.
-- explicitSig explicitSig
```

-- Note: only for explicit signatures, i.e. MES version v3 and v4.

- -- algorithm subjectPublicKeyInfo.algorithmIdentifier
- -- email X.509 email extension (version must be v3)

END

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