Pseudorandom Correlation Generators:

Secure Computation with Silent Preprocessing

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Technion

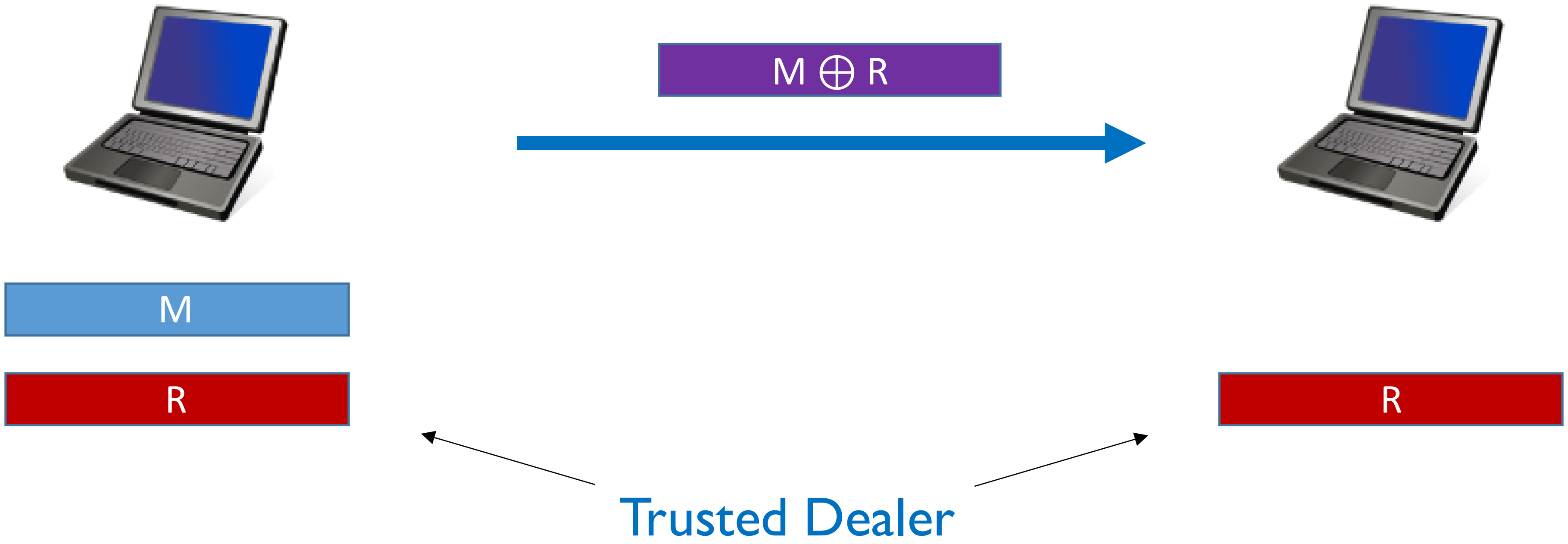
Based on joint works with Elette Boyle, Geoffroy Couteau, Ronald Cramer,
Ivan Damgård, Niv Gilboa, Lisa Kohl, Peter Rindal, and Peter Scholl

This talk

- Motivation: Secure computation with silent preprocessing
- Primitive: Pseudorandom Correlation Generator (PCG)
- Survey of PCG constructions

Secure Communication from common randomness

[Shannon 1944]



$M \oplus R$

M

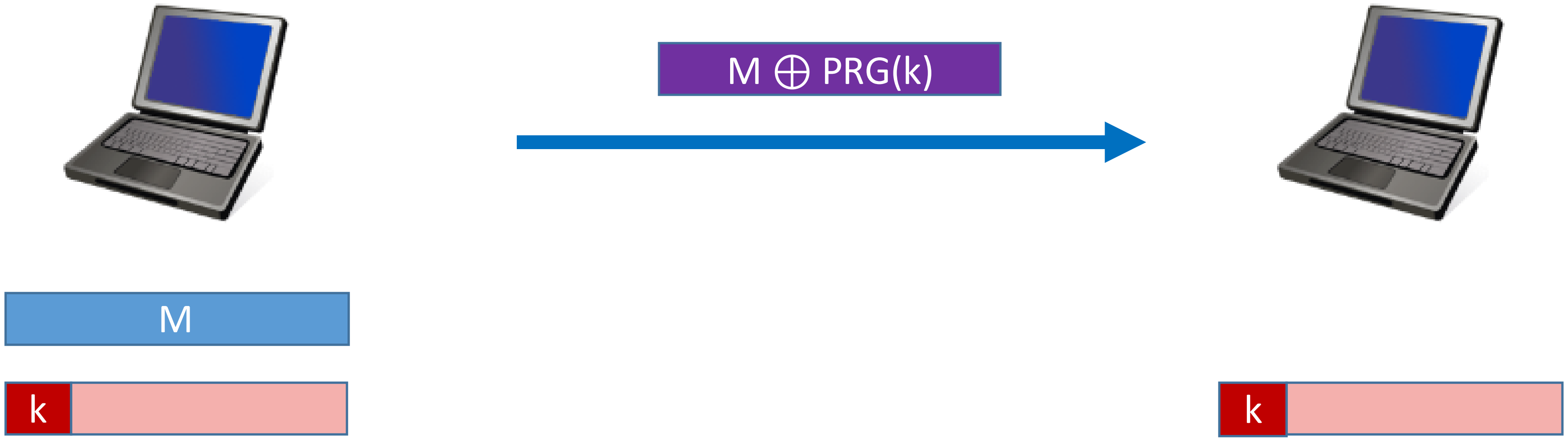
R

R

Trusted Dealer

Secure Communication from pseudorandomness

[Blum-Micali 1982, Yao 1982]



$M \oplus \mathrm{PRG}(k)$

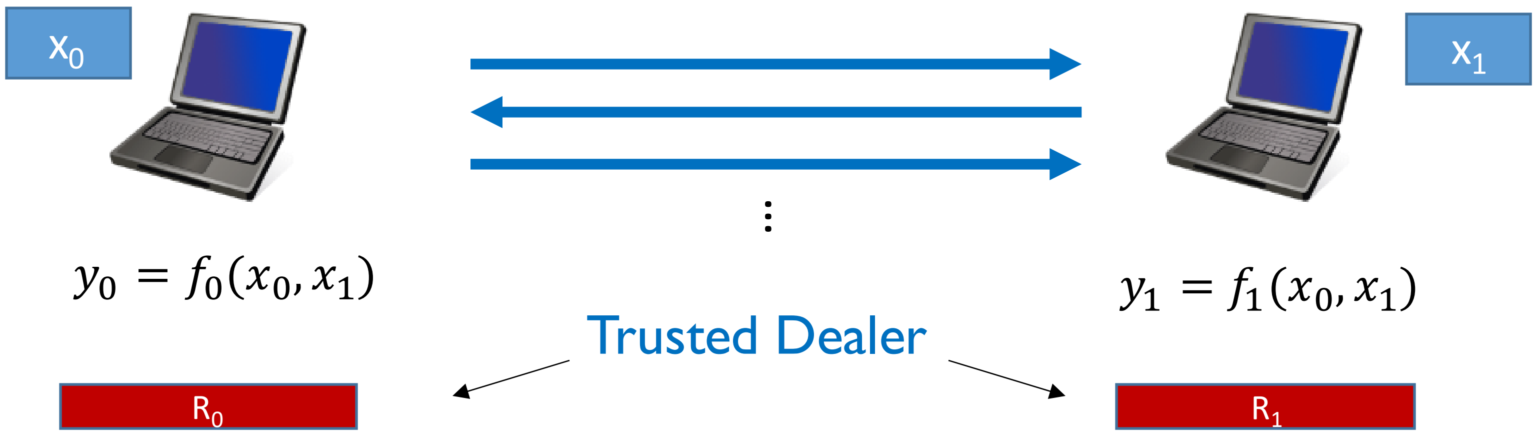
M

k

k

Secure Computation from correlated randomness

[Beaver 1995]



x_0

x_1

$y_0 = f_0(x_0, x_1)$

$y_1 = f_1(x_0, x_1)$

Trusted Dealer

R_0

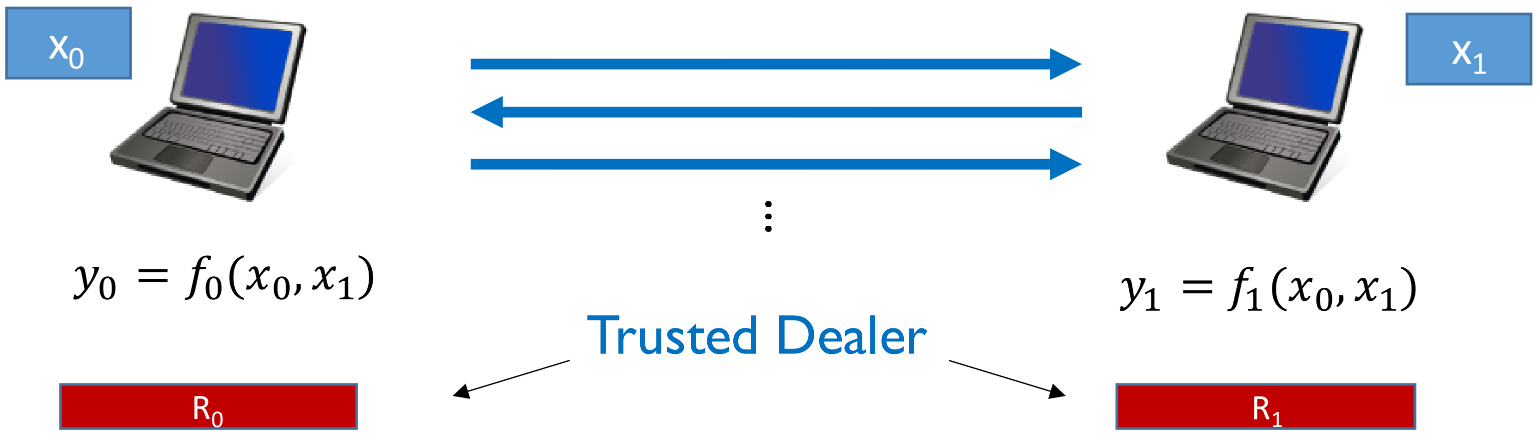
R_1

- Information-theoretic security
- Constant computational overhead

[Bea95, Bea97, IPS08, BDOZ11, BIKW12, NNOB12, DPSZ12, IKMOP13, DZ13, DLT14, BIKK14, LOS14, FKOS15, DZ16, KOS16, DNNR17, C18, BGI19, …]

Secure Computation from correlated randomness

[Beaver 1995]



x_0

x_1

$y_0 = f_0(x_0, x_1)$

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Trusted Dealer

R_0

R_1

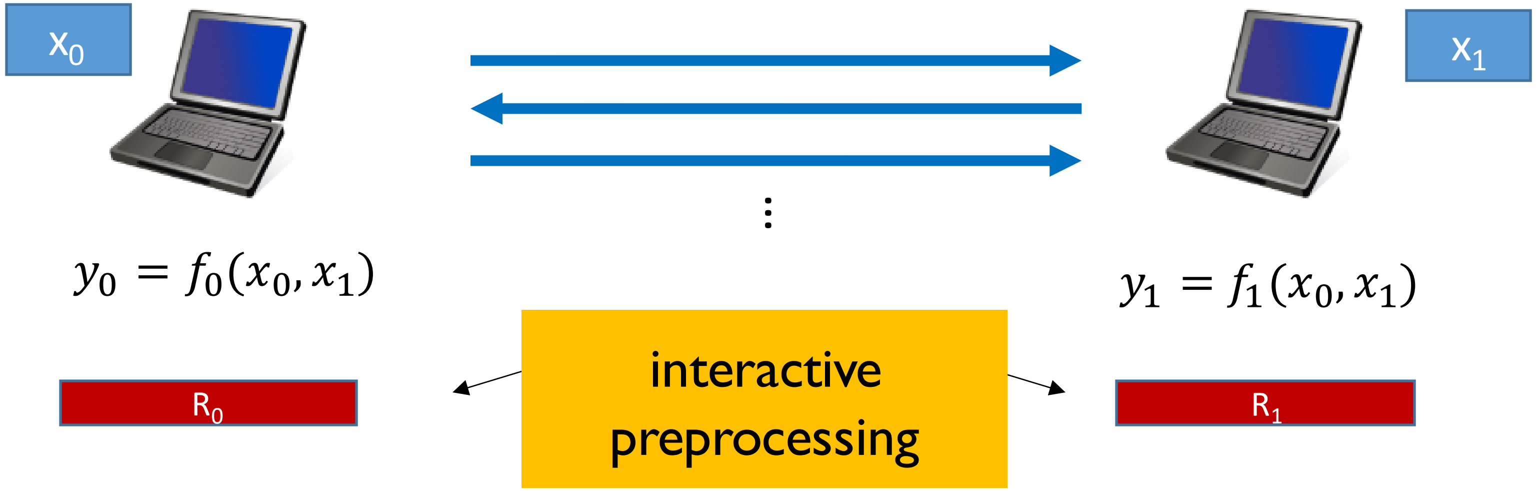
- Information-theoretic security
- Constant computational overhead

Useful correlations:

OT, OLE,VOLE, (authenticated) multiplication triples,
one-time truth-table, **multi-party** linear correlations

Secure Computation from correlated randomness

[Beaver 1995]



x_0

x_1

$y_0 = f_0(x_0, x_1)$

$y_1 = f_1(x_0, x_1)$

R_0

interactive preprocessing

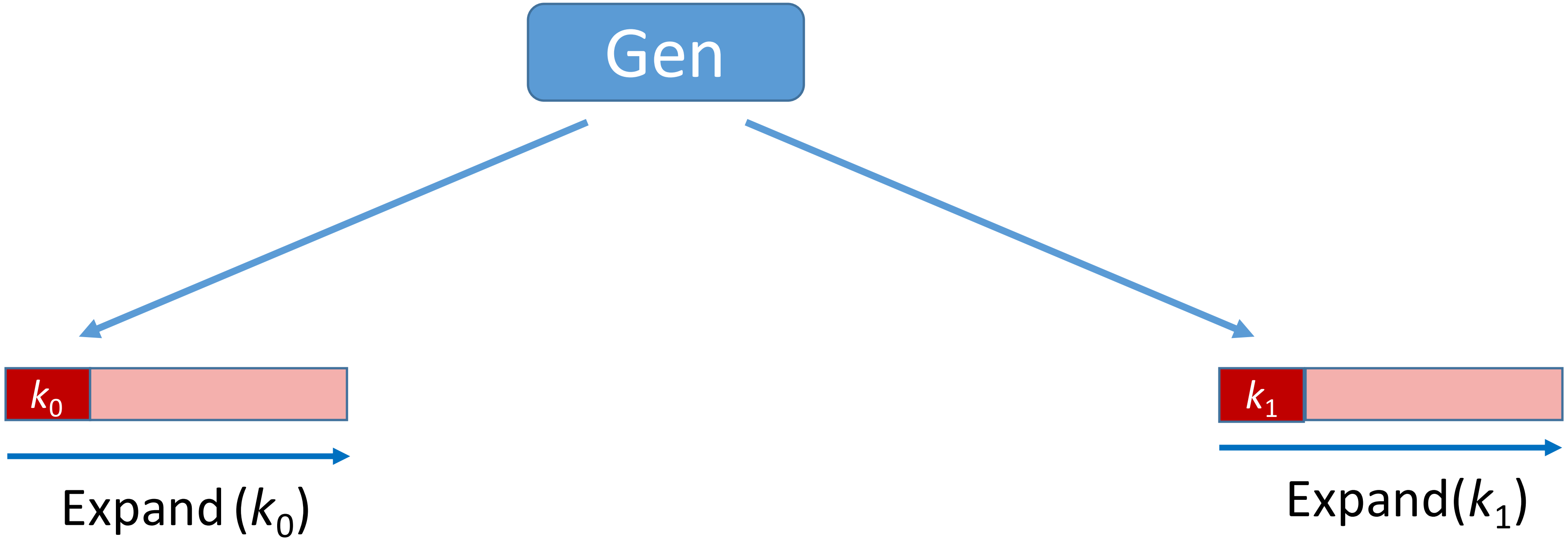
R_1

Dominates the total cost

Secure Computation
from correlated pseudorandomness?

Pseudorandom Correlation Generator (PCG)

[BCGI18, BCGIKS19]

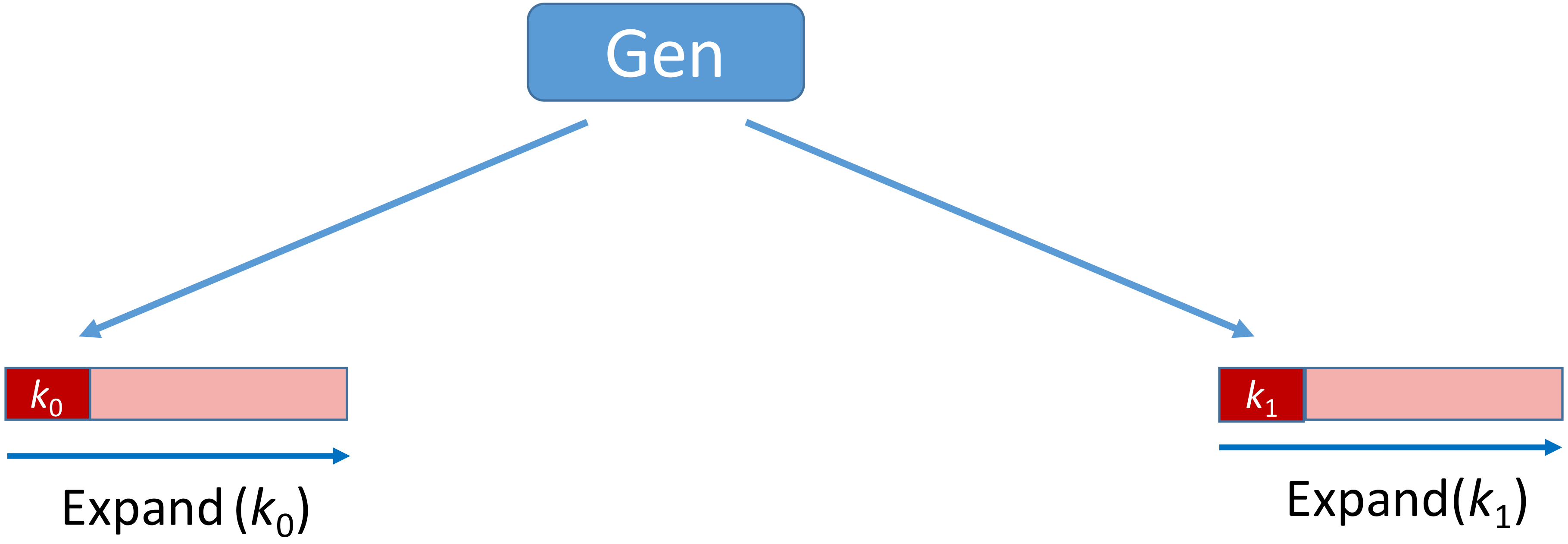


Target correlation: (R_0, R_1)

$$\left(\mathrm{Expand}(k_0), \mathrm{Expand}(k_1)\right) \approx (R_0, R_1)$$

Pseudorandom Correlation Generator (PCG)

[BCGI18, BCGIKS19]



How do we define security against insiders?

PCG: Security Definition

- Take I: Real = $(k_0, \mathrm{Expand}(k_1)) \approx (\mathrm{Sim}(R_0), R_1)$ = Ideal

Securely realizing ideal correlation functionality

Good for all applications

Not realizable even for simple correlations

PCG: Security Definition

- Take I: Real = $(k_0, \mathrm{Expand}(k_1)) \approx (\mathrm{Sim}(R_0), R_1)$ = Ideal
- Take II: Real = $(k_0, \mathrm{Expand}(k_1)) \approx (k_0, [R_1 \mid R_0 = \mathrm{Expand}(k_0)])$

Securely realizing “corruptible” correlation functionality

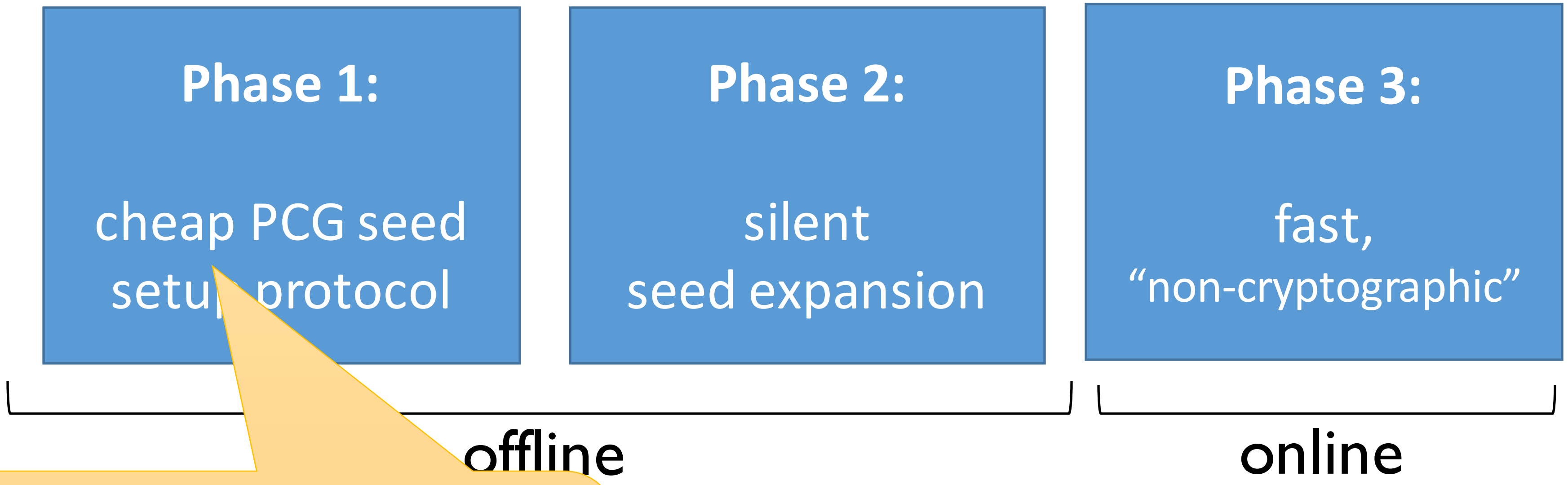
Good for natural applications

Realizable for useful correlations

PCG: Security Definition

- Take I: Real = $(k_0, \mathrm{Expand}(k_1\ )) \approx (\mathrm{Sim}(R_0), R_1)$ = Ideal

MPC with Silent Preprocessing



Phase 1:

cheap PCG seed setup protocol

Phase 2:

silent seed expansion

Phase 3:

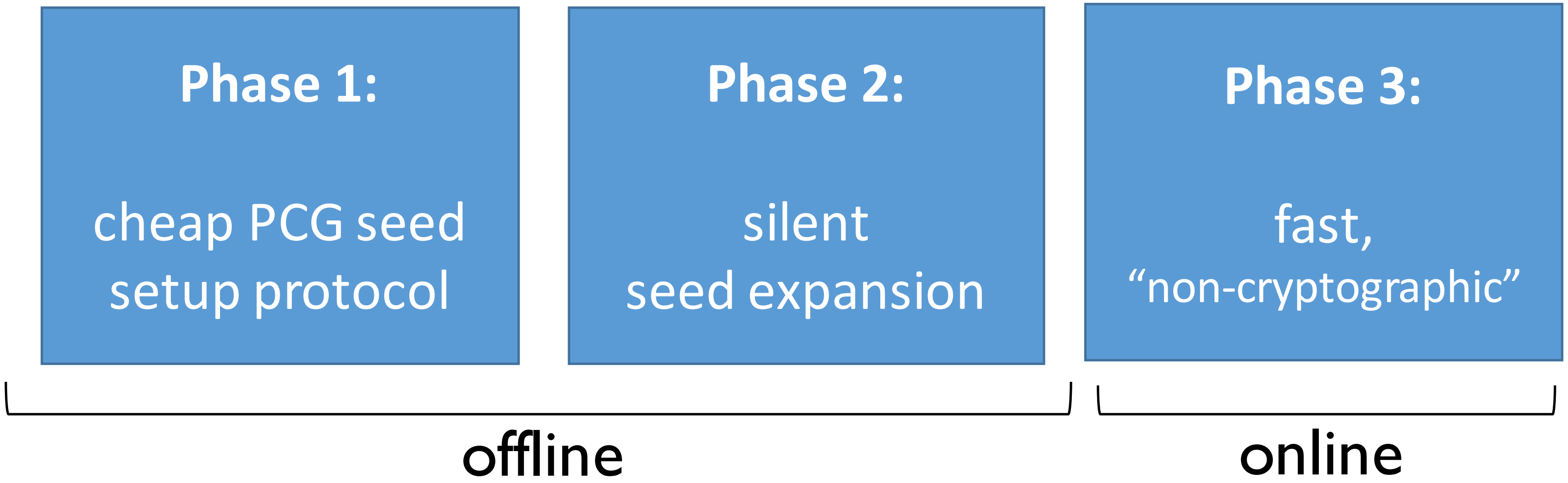
fast, "non-cryptographic"

offline

online

- ✓ Ad-hoc future interactions
- ✓ Hiding communication pattern
- ✓ Hiding future plans

MPC with Silent Preprocessing



Phase 1:

cheap PCG seed
setup protocol

Phase 2:

silent
seed expansion

Phase 3:

fast,
“non-cryptographic”

offline

online

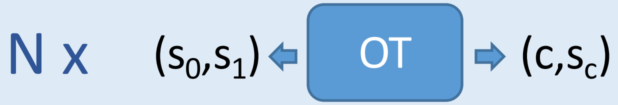
- Improved overall communication
- Near-optimal online computation
- Active security with **vanishing** amortized cost

Useful target correlations: 3+ parties

Linear n-party correlations	$(R_0, \ldots, R_{n-1}) \in_R$ Linear space V N x deg-t Shamir of random secret N x additive shares of 0	VSS, honest-majority MPC Proactive secret sharing Secure aggregation

Useful target correlations: 2+ parties

Oblivious transfer (OT)

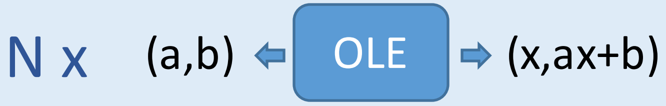

N x (s_0,s_1) ⇦ OT ⇨ (c,s_c)

2PC of Boolean circuits

GMW-style, passive:

2 x bit-OT + 4 comm. bits per AND

Oblivious Linear-function Evaluation (OLE)

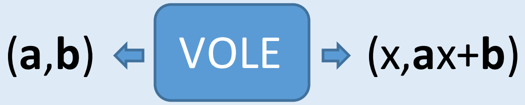

N x (a,b) ⇦ OLE ⇨ $(x,ax+b)$

2PC of Arithmetic circuits

GMW-style, passive:

2 x OLE + 4 ring elements per MULT

Vector OLE (VOLE)


$(\mathbf{a},\mathbf{b})$ ⇦ VOLE ⇨ $(x,\mathbf{a}x+\mathbf{b})$

2PC of scalar-vector product

Zero knowledge

PSI

Useful target correlations: 2+ parties

Authenticated Multiplication Triples	$([a_i],[b_i],[c_i],\ [\alpha a_i],[\alpha b_i],[\alpha c_i])$ $c_i=a_ib_i$	2PC of Arithmetic circuits SPDZ-style, active
Truth-tables	Randomly shifted, Secret-shared TT	2PC of “unstructured” functions
Additive	R0+R1 = R	Generalizes all the above

Current PCG Landscape

“Obfustopia”	iO	General [HW15, HIJKR16]
“Homomorphia”	LWE+	Additive [DHRW16, BCGIKS19]
“Cryptomania”	DDH,LWE	Low-depth [BCGIO17, BCGIKS19]
“Lapland”	LPN	VOLE, OT [BCGI18, BCGIKS19]
	Ring-LPN	OLE, (Auth.) Triples [BCGIKS20a]
	VD-LPN	PCF for VOLE, OT [BCGIKS20b]
“Minicrypt”	PRG	Linear multi-party [GI99, CDI05]
		Truth table [BCGIKS19]

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“Lapland”	LPN Ring-LPN VD-LPN	Constant-degree additive (poly(N) expansion time)
“Minicrypt”	PRG	Linear multi-party [GI99, CDI05] Truth table [BCGIKS19]

Good concrete efficiency?

“Obfustopia”	iO	General [HW15, HIJKR16]
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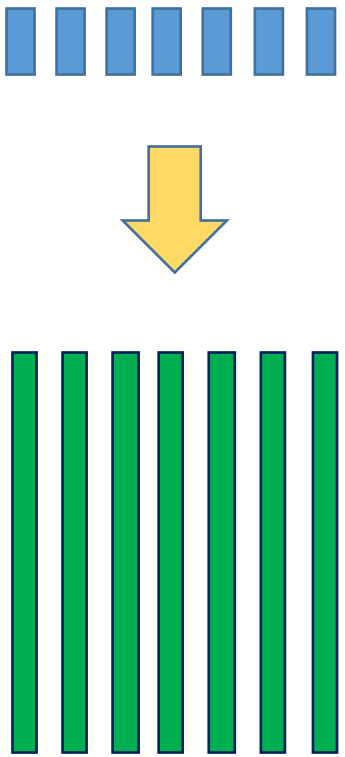
		Truth table [BCGIKS19]

Pseudorandom secret sharing (PRSS)

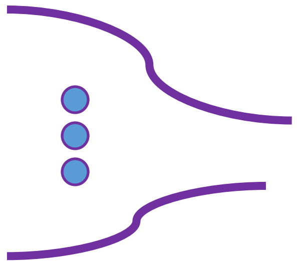
“Minicrypt” PRG Linear multi-party [GI99, CDI05]

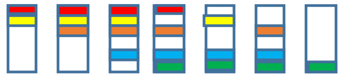


~ 0.3 KB seeds

~ 0.1 second

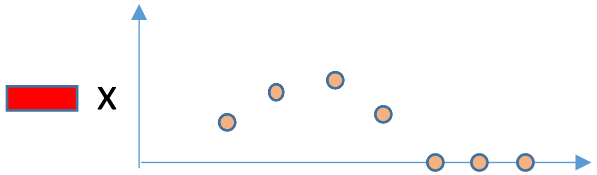
10^6 x deg-3 Shamir





deg-t share vectors

$\binom{n}{t}$ replicated PRG seeds



general linear

seed per min-support codeword

only efficient when $\binom{n}{t}$ is “[illegible]”

Additive shares of 0: $\binom{n}{2}$ seeds

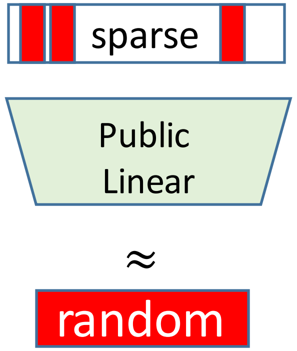
LPN-based PCGs: Tools

"Lapland"	LPN	VOLE, OT [BCGI18, BCGIKS19]
	Ring-LPN	OLE, (Auth.) Triples [BCGIKS20a]

(Dual) LPN

(Quasi)linear time

Peter's talk



Also over large fields / rings

Compressing secret-shared (N,w) sparse vector

Distributed Point Function
Function Secret Sharing
[GI14,BGI15,BGI16]

OLE, Triples
Truth-table, PCF

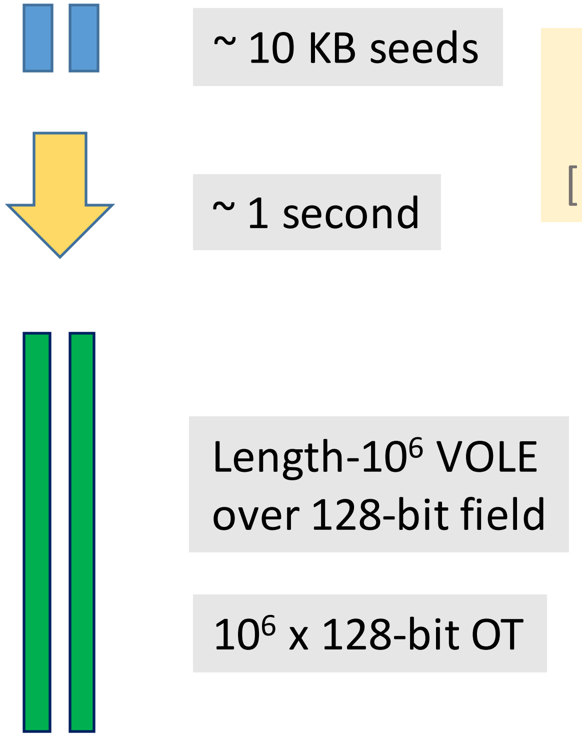
Puncturable PRF
[KPTZ13,BW13,BGI14]

VOLE, OT

GGM-style PRG tree
wlog(N) PRG seeds
O(N) x PRG calls expansion

LPN-based PCGs: VOLE and OT

“Lapland” LPN VOLE, OT [BCGI18, BCGIKS19]



~ 10 KB seeds

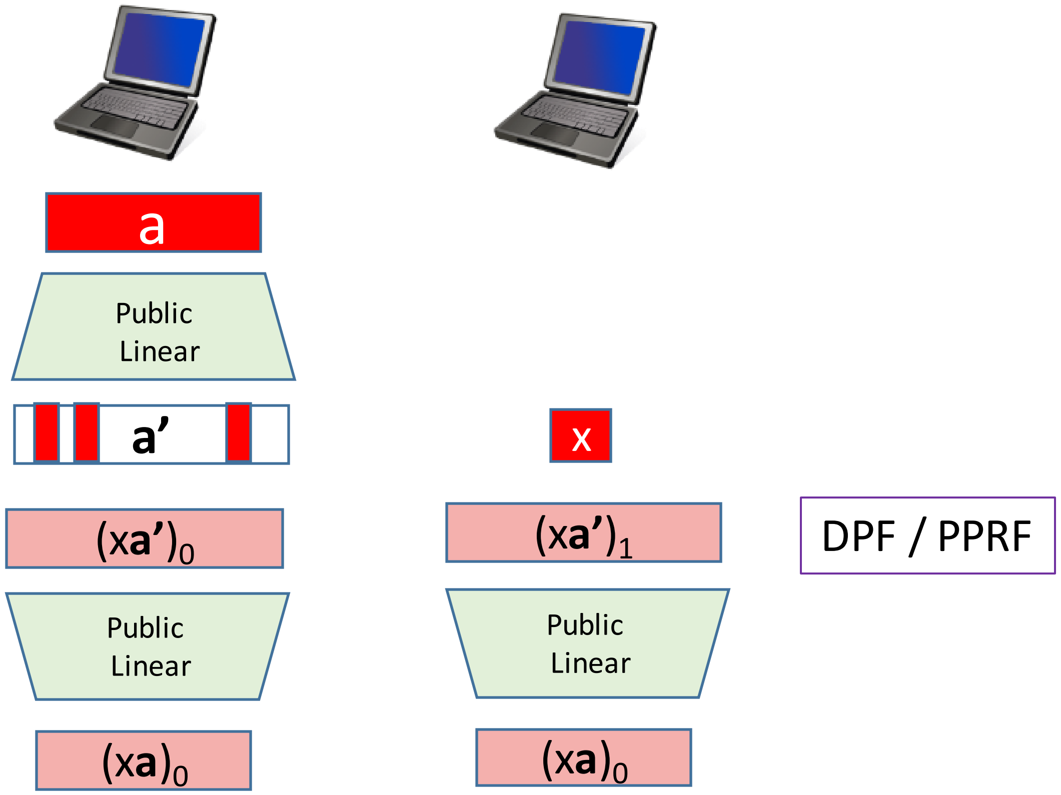
~ 1 second

Length-10^6 VOLE over 128-bit field

10^6 x 128-bit OT

~ 100 KB 2-round seed generation [BCGIKRS19,SGRR19]

Peter’s talk



a

Public Linear

a’

x

$(xa')_0$

$(xa')_1$

DPF / PPRF

Public Linear

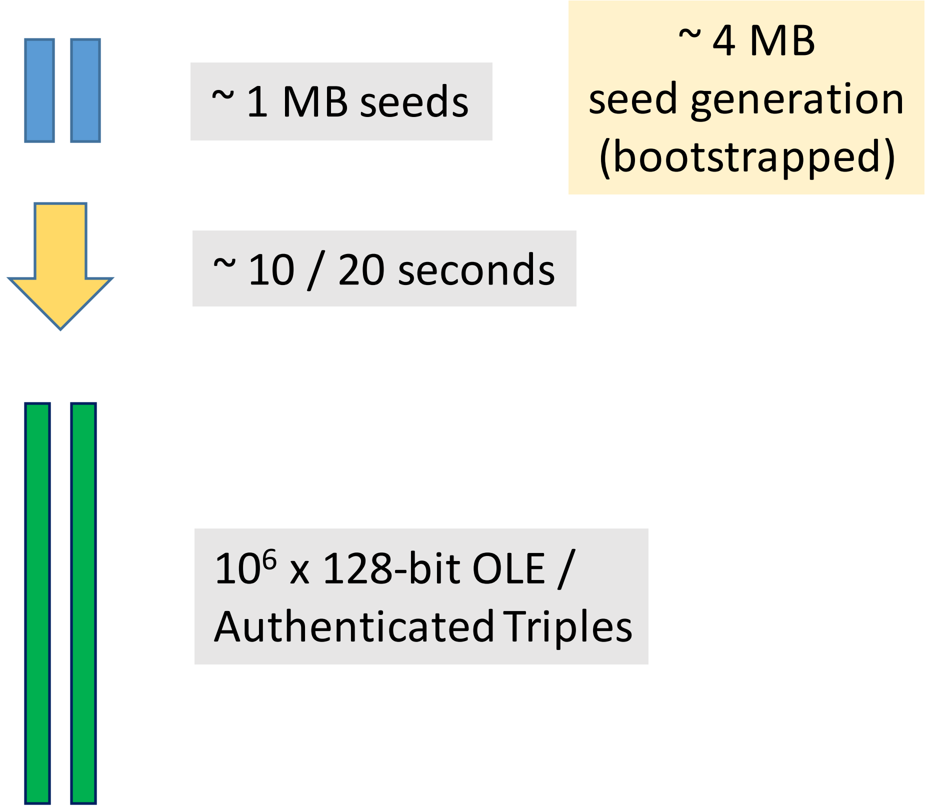
Public Linear

$(xa)_0$

$(xa)_0$

LPN-based PCGs: OLE and Triples

“Lapland” Ring-LPN VOLE, OT [BCGIKS20a]



~ 1 MB seeds

~ 4 MB
seed generation
(bootstrapped)

~ 10 / 20 seconds

10^6 x 128-bit OLE /
Authenticated Triples

Non-silent alternatives:
Overdrive [KPR18]
Leviosa [HIVM19]

x100-x1000 communication
comparable run time

Further Research

Better PCGs

- More correlations?
  - Garbled circuits, N x truth-tables, N x PCG seeds, …
- Multi-party variants
  - Shamir with t=n/2, authenticated triples
- Smaller seeds, faster expansion and seed generation

Better understanding of LPN-style assumptions

- Which codes?
- Which noise patterns?
- LPN vs. LWE

Better PCFs

The End

- Questions?